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DB-003-001544

Seat No. ________

B. Sc. (Sem. V) (CBCS) Examination

March – 2022

Statistics : Paper - 503

(Statistical Inference)

(Old Course)

Faculty Code : 003

Subject Code : 001544

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) Q.1 carry 20 marks and Q.2 and Q.3 carry 25 marks each.

(2) Right hand side figures shows marks of that questions.

(3) Student can use their own certified calculator.

1 Filling the blanks and short questions : **20**

(1) Estimation is possible only in case of a ________

(2) A single value of an estimator for a population parameter θ is called its ________ estimate.

(3) The difference between the expected value of an estimator and the value of the corresponding parameters is known as ________

(4) $\sum \frac{X_i}{n}$ *for* $i = 1, 2, 3, ..., n$ is a ________ estimator of population mean.

(5) For mean square error to be minimum, bias should be _________

(6) An estimator of $v_\theta(T_n)$ which attains lower bound for all θ is known as _________

(7) If $f(x;\theta)$ is a family of distributions and $h(x)$ is any statistic such that $E[h(x)]=0$, then $f(x;\theta)$ is called _________

(8) If a random sample $x_1, x_2, x_3, \ldots x_n$ is drawn from a population $N(\mu, \sigma^2)$, the maximum likelihood estimate of σ^2 is _________

(9) For a Gama (x, α, λ) distribution with λ known, the maximum likelihood estimate of α is _________

(10) Maximum likelihood estimate of the parameter θ of the distribution $f(x,\theta)=\frac{1}{2}e^{-|x-\theta|}$ is _________

(11) _________ is an unbiased estimator of p^2 in Binomial distribution.

(12) The estimate of the parameter λ of the exponential distribution $\lambda e^{-\lambda x}$ by the method of moments is _________

(13) For a rectangular distribution $\frac{1}{(\beta-\alpha)}$, the maximum likelihood estimates of α and β are _________ and _________ respectively.

(14) If $x_1, x_2, x_3, \ldots x_n$ is a random sample from an infinite population and S^2 is defined as $\frac{\sum(x_i-\bar{x})^2}{n}, \frac{n}{n-1}S^2$ is an _________ estimator of population variance σ^2.

(15) Let there be a sample of size n from a normal population with mean μ and variance σ^2. The efficiency of median relative to the mean is ________

(16) Minimum Chi-square estimators are not necessarily ________.

(17) If a function $f(t)$ of the sufficient statistics $T = t(x_1, x_2, x_3, ... x_n)$ is unbiased for $\tau(\theta)$ and is also unique, this is the ________

(18) If sufficient estimator exists, it is function of the ________

(19) Sample mean is an ________ and ________ estimate of population mean.

(20) If T_1 and T_2 are two MVU estimator for $T(\theta)$, then ________.

2 (A) Write the answer any **three** : (Each 2 marks) **6**

(1) Define Consistency

(2) Define Sufficiency

(3) Define Uniformly Most Powerful Test (UMP test)

(4) Define ASN function of SPRT

(5) Obtain likelihood function of Negative Binomial distribution.

(6) Find the Cramer Rao lower bound of variance of unbiased estimator of parameter of the probability distribution $f(x, \theta) = \theta e^{-\theta x}$

(B) Write the answer any **three** : (Each 2 marks) **9**

(1) Obtain unbiased estimator of θ for $f(x,\theta)=\frac{1}{\theta}e^{-\left(\frac{x}{\theta}\right)}$ where $0\le x\le\infty$

(2) Prove that $E\left(\frac{\partial \log L}{\partial\theta}\right)^2=-E\left(\frac{\partial^2 \log L}{\partial\theta^2}\right)$

(3) Obtain MVUE of parameter θ for Poisson distribution. Also obtain its variance.

(4) Obtain estimator of θ by method of moments in the following distribution $f(x;\theta)=\theta e^{-\theta x}$; where $0\le x\le\infty$

(5) Obtain Operating Characteristic (OC) function of SPRT.

(6) Give a random sample $x_1,x_2,x_3,\ldots x_n$ from distribution with p.d.f. $f(x;\theta)=\frac{1}{\theta};0\le x\le\theta.$ Obtain type-I and type-II errors. Also obtain power of the test for testing $H_0:\theta=1$ against $H_1:\theta=2$ where $c=\{x;x\ge 0.5\}.$

(C) Write the answer any **two** : (Each 5 marks) **10**

(1) State Neyman-Pearson lemma and prove it.

(2) If $x_1,x_2,x_3,\ldots x_n$ random sample taken from distribution with mean θ and σ^2 variance then $(i)\ t_1=\frac{\sum x_i}{n+1}\ (ii)\ t_2=\frac{\sum x_i}{n-1}$ Is a consistent of θ check it?

(3) Estimate α and β in the case of Gamma distribution by the method of moments

$$f(x;\alpha,\beta)=\frac{\alpha^{\beta}}{\Gamma\beta}e^{-\alpha x}x^{\beta-1};x\geq 0,\alpha\geq 0$$

(4) Obtain OC function for SPRT of Binomial distribution for testing $H_0:p=p_0$ against $H_1:p=p_1(>p_0)$

(5) Obtain Likelihood Ration Test:

Let $x_1,x_2,x_3,...x_n$ random sample taken from $N\left(\mu,\sigma^2\right)$ To test $H_0:\sigma^2=\sigma_0^2$ against $H_1:\sigma^2\neq\sigma_0^2$

3 (A) Write the answer any **three** : (Each 2 marks) **6**

(1) Define Unbiasedness

(2) Define Efficiency

(3) Define Minimum Variance Bound Estimator (MVBE)

(4) Define Most Powerful Test (MP test)

(5) Obtain likelihood function of Laplace distribution.

(6) Find the Cramer Rao lower bound of variance of unbiased estimator of parameter of the probability distribution $f(x,\theta)=\theta x^{\theta-1}$

(B) Write the answer any **three** : (Each 3 marks) **9**

(1) Let $x_1,x_2,x_3,...x_n$ be random sample taken from $N\left(\mu,\sigma^2\right)$ then find sufficient estimator of μ and σ^2.

(2) Obtain an unbiased estimator of population mean of χ^2 distribution.

(3) Obtain MLE of parameter θ of $f(x,\theta) = \theta^x (1-\theta)^{(1-x)}$ where $x = 0,1$ and $0 \le \theta \le 1$

(4) If A is more efficience than B then prove that $Var(A) + Var(B - A) = Var(B)$

(5) Give a random sample $x_1, x_2, x_3, ... x_n$ from the distribution with p.d.f. $f(x,\theta) = \theta e^{-\theta x}$, where $x > 0$. Using the Neyman-Pearson Lemma to obtain the best critical region for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$

(6) Let p be the probability that coin will fall head in a single toss in order to test $H_0 ; p = \frac{1}{2}$ against $H_1 ; p = \frac{3}{4}$. The coin is tossed 5 times and H_0 is rejected if more than 3 head are obtained. Find the probability of type-I error, type-II error and power of test.

(C) Write the answer any **two** : (Each 5 marks) **10**

(1) State Crammer-Rao inequality and prove it.

(2) Obtain MVBE of σ^2 for Normal distribution $\left(0, \sigma^2\right)$.

(3) Obtain the MLE of α and β for random sample from the exponential population $f(x_i, \alpha, \beta) = y_0 e^{-\beta(x-\alpha)}$ where $\le x \le \infty, \beta > 0$. y_0 being a constant.

(4) Let $x_1, x_2, x_3, \ldots x_n$ be random sample from the p.d.f.

$$f(x, p) = \frac{1}{(1-q^3)} \binom{3}{x} p^x q^{3-x} \quad \text{where} \quad x = 0, 1, 2, 3$$

Estimate parameter of p by the method of moment.

(5) Construct SPRT of Poisson distribution for testing $H_0 : \lambda = \lambda_0$ against $H_1 : \lambda = \lambda_1 (> \lambda_0)$. Also obtain OC function of SPRT.
